Non-Stationary Random Vibration Parametric Modelling and Identification: methods and applications

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Abstract

Non-stationary random vibration signals exhibit time-dependent characteristics and require non-parametric models and corresponding identification methods. The focus is on parametric models and identification methods, which are classified according to the type of model parameter temporal evolution postulated: unstructured, stochastic and deterministic. The relative model characteristics and the primary identification methods within each class are discussed, along with model analysis issues. Three application case studies are then briefly considered: (i) Output-only identification of the non-stationary dynamics of a laboratory bridge-like structure with moving mass, (ii) non-stationary modelling of the El Centro earthquake ground motion signal, and (iii) output-only identification of non-stationary wind turbine dynamics under normal operating conditions.

Keywords: non-stationary vibration, time-varying dynamics, parametric models, time-varying autoregressive moving average (TV-ARMA) models, time-frequency distributions, bridge-like structures, earthquake ground motion, wind turbine structures.

1. Introduction

Non-stationary random vibration signals exhibit time-dependent characteristics and require non-parametric models and corresponding identification (estimation) methods [1-6]. They are frequently encountered in engineering practice as the result of non-stationary excitation (such as earthquake excitation, atmospheric turbulence, and so on) or Time–Varying (TV) structural dynamics (as in traffic–excited bridges, robotic devices, deployable structures, crane systems, wind turbines, and so on) operating under random excitation.

There are two main approaches to modelling non-stationary random vibration signals from a measured data record: (i) the non-parametric, which includes spectrogram and other time-frequency or wavelet representations; and (ii) the parametric, which includes the Time–dependent AutoRegressive Moving Average (TARMA) and corresponding State Space (SS) models [1-7]. The present paper focuses on parametric models and their identification. These are generally known to offer increased accuracy and resolution, while also achieving enhanced parsimony (model compactness) [2,8,9].

Obtaining parametric models from a measured signal record constitutes the model identification problem. Identified models may be used for analyzing and understanding the non-stationary signal characteristics and predicting its future evolution. They may be also used for the output-only modelling of the underlying TV structural dynamics for purposes of improved understanding, analysis, structural modification, optimization, Structural Health Monitoring (SHM), and control.

In treating these models it is necessary to consider notions such as the Time-Varying AutoCovariance Function (TV-ACF) and its Fourier Transform which is the Power Spectral Density (PSD). Oftentimes in practice, some sort of TV-PSD function is preferable, and several attempts for defining “appropriate” such functions have been made. These include the “evolutive” and “frozen” TV-PSDs. The former may be more appropriate for describing the time evolution of the vibration signal characteristics, while the latter may be more appropriate for describing the underlying TV structural dynamics [2,7].

The aim of this article is to provide a brief snapshot of the current state--of--the--art on parametric methods and three application case studies: (i) Output-only identification of the TV dynamics of a laboratory bridge-like structure with moving mass (simulating a bridge with moving vehicle), (ii) non-stationary modelling of an earthquake ground motion signal, and (iii) the output-only identification of TV wind turbine dynamics under normal operating conditions.

2. Concise summary of parametric non-stationary models & identification methods

Parametric non-stationary models may be of the time-dependent ARMA (TARMA) or time–dependent State Space (SS) forms [2,10,11]. A TARMA(nφ,nθ) model, with nφ, nθ designating its Autoregressive (AR) and Moving Average (MA) orders, respectively, is of the form (the scalar case is considered):

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\[
\begin{align*}
x[t] + \sum_{i=1}^{n_a} a_i[t]x[t-i] &= w[t] + \sum_{j=1}^{n_c} c_j[t]w[t-j] \\
w[t] &\sim NID(0, \sigma_w^2[t])
\end{align*}
\]

with \( t \) designating normalized discrete time (sampling period \( T_s \)), \( x[t] \) the non-stationary vibration signal modelled, \( w[t] \) an unobservable uncorrelated (white) non-stationary signal generating \( x[t] \) (also representing the optimal one-step-ahead prediction error and referred to as the residual signal). \( w[t] \) is characterized by zero mean and TV variance \( \sigma_w^2[t] \). \( a_i[t] \), \( c_j[t] \) are the model’s TV AR and MA parameters, respectively. \( NID(\cdot, \cdot) \) stands for normally independently distributed random variables with the indicated mean and variance. A time–dependent SS(\( n \)) model is of the form (for the scalar vibration signal case):

\[
\begin{align*}
x[t] &= C[t]z[t] + w[t] \\
w[t] &\sim NID(0, \sigma_w^2[t])
\end{align*}
\]

with \( n \) designating model order, \( z[t] \) (\( n \times 1 \)) the state vector and \( A, C, K \) proper matrix/vector quantities (bold face symbols indicate matrix/vector quantities). Depending on the nature of the mathematical structure imposed on the time evolution of their parameters, the above models may be classified as: (i) unstructured parameter evolution, (ii) stochastic parameter evolution, and (iii) deterministic parameter evolution.

2.1. Unstructured parameter evolution models

These models impose no mathematical structure on the time evolution of their parameters, which are thus free to change with time. They are thus directly parametrized in terms of their TV parameters \( a_i[t] \), \( c_j[t] \), \( \sigma_w^2[t] \) which need to be specified for each time instant. A specific model structure includes the AR and MA orders \( n_a, n_c \), that is \( M_{UR} = [n_a, n_c] \). These models may be estimated via short–term (locally stationary) approaches by applying well known stationary techniques within a short time window that slides over the data record [12]. Alternatively, recursive approaches that update the parameter vector each time a new sample of the vibration signal is processed may be also used [13-15]; a prime example being the Recursive Maximum Likelihood TARMA (RML–TARMA) method [2,14].

2.2. Stochastic parameter evolution models

These models impose stochastic structure on the time evolution of their parameters. Typically, these are assumed to be serially (in time) autocorrelated random variables, with their evolution being subject to stochastic smoothness constraints (typically these take the form of integrated stochastic models). The models of this class are referred to as Smoothness Priors TARMA (SP–TARMA) models [5]. The degree of smoothness of the time evolution of each individual parameter is controlled by the corresponding driving white noise variance, and increases for decreasing variance. A specific SP–TARMA model structure is thus defined by the model orders \( n_a, n_c \) and the smoothness constraints order \( \kappa \), that is \( M_{SP} = [n_a, n_c, \kappa] \). The estimation of SP–TARMA models is achieved via Kalman Filtering in the pure AR case, while Extended Kalman Filter (EKF) or Extended Least Squares (ELS) type algorithms are used in the full ARMA case [2,5].

2.3. Deterministic parameter evolution models

These models impose deterministic structure on the time evolution of their parameters. This is achieved by postulating model parameters as deterministic functions of time belonging to specific functional subspaces defined via selected basis functions [2,8,9,16-21]. Such models are then referred to as Functional Series TARMA (FS–TARMA) models. An FS–TARMA model is thus parameterized in terms of AR, MA, and innovations variance projection coefficients \( a_{0j}, c_{0j}, \sigma_j \), respectively, while a specific model structure \( M_{FS} \) is defined by the model orders \( n_a, n_c \), and the AR, MA, and innovations variance functional subspaces \( F_{AR}, F_{MA} \) and \( F_{\sigma_j} \), respectively. FS–TARMA models achieve high parsimony (compactness of representation). Moreover, through proper selection of their functional subspaces, they are capable of representing various types of evolution in the dynamics, including “slow”, “fast” or “discontinuous” [2,9,21].

FS–TARMA model identification may be based on the Maximum Likelihood or regression–type methods [2,7,9,21]. The former requires good initial values for correct convergence to the global extremum of the cost function, and the same is true for the latter category in case of non–linear regression (Prediction Error) type estimators (especially in the full ARMA case). For this reason a number of relaxation methods have been introduced. The Two Stage Least Squares (2SLS) [2,17], Polynomial–Algebraic (P–A) [20] and the recently introduced Multi–Stage (MS) methods [9] are most prominent, although there are several selections and variations available – see [2,9,21]. The model structure selection problem is usually based on integer optimization employing genetic algorithms or on suboptimal search schemes [2,9,21]. The asymptotic properties of various estimators are studied in [22].

An important recent development in this area is the postulation of FS–TARMA models with complex exponential or spline basis functions, accompanied by the development of a Separable Nonlinear Least Squares (SNLS) method which achieves simultaneous estimation of the model coefficients of projection and the basis functions themselves [9]. This method drastically simplifies the identification procedure; results from application case studies are very promising.
3. Estimated model analysis: the notions of frequency response and power spectral density

As is well known, there exists no frequency response concept for TV linear systems. Yet, there are various attempts to define “similar” approximate concepts, mainly for practical purposes and for facilitating analysis. One such definition is the “evolutive” time-varying Frequency Response Function (“evolutive” TV-FRF):

$$H(e^{j\omega \tau}, t) = \frac{\text{response of the system to } e^{j\omega \tau}t}{e^{j\omega \tau}t} = \sum_{i=0}^{\infty} h[t, t - i] \cdot e^{-j\omega \tau}i$$ \hspace{1cm} (3)

where \(j\) designates the imaginary unit, \(\omega\) frequency in rad per unit time, and \(h[t, t]\) the underlying TV impulse response function. Another definition considers the structural dynamics “frozen” at a particular time instant \(\tau = \tau_n\); hence the dynamics at that time instant are represented by the impulse response function \(h[t, \tau_n]\). A corresponding TV-FRF may be then defined as (changing \(\tau_n\) to \(\tau\); the “frozen” TV-FRF):

$$H(e^{j\omega \tau}, t) = \sum_{i=0}^{\infty} h[i, t] \cdot e^{-j\omega \tau}i$$ \hspace{1cm} (4)

Similarly, a TV-PSD concept is sought. Several forms of time-dependent PSDs have been proposed, but lack the important properties and physical significance of the stationary PSD [17, 23–25]. Corresponding to the above FRF concepts are:

The Melard-Tjostheim (“evolutive”) TV-PSD:

$$S_{MT}(\omega, t) \triangleq \sum_{i=0}^{t} h[i, t] \cdot \sigma_{\omega}^2[t] \cdot e^{j\omega \tau}t$$

The “frozen” TV-PSD or Grenier’s Rational Relief:

$$S_{F}(\omega, t) \triangleq \sum_{i=0}^{\infty} h[i, t] \cdot \sigma_{\omega}^2[t] \cdot e^{j\omega \tau}t$$

In cases of “sufficiently” slowly varying dynamics, the actual TV dynamics may be approximated by the temporal sequence of the “frozen” dynamics.

4. Application case studies

Three application case studies are briefly presented: (i) The output-only identification of a bridge-like structure with moving mass used to simulate a vehicle passing over a bridge; (ii) the modeling of the El Centro earthquake ground motion signal; and (iii) the output-only identification of wind turbine structural dynamics under normal operating conditions.

4.1. Output-only identification of a bridge-like structure with moving mass

The schematic diagram of the experimental setup is presented in Figure 1. A steel beam is clamped close to its both ends on two vertical stands. A steel cylindrical mass slides on it, being pulled by a DC motor at a selected speed. The beam-mass system is characterized by continuously varying mass distribution. The beam is subject to a random, zero mean and approximately white Gaussian force excitation exerted via an electromechanical shaker equipped with a stinger. The vertical vibration is measured at three locations via piezoelectric accelerometers, but only the response at location 3 (Figure 1) is presently used. The measured vibration signal is conditioned and driven into a DAQ module (sampling frequency \(f_s = 128\ Hz\), signal length \(N = 10\,113\) samples; Figure 2).

![Schematic diagram of the bridge like-structure with moving mass experiment](image1)

![Bridge-like structure with moving mass](image2)

Non-stationary TARMA models of the random vibration signal are estimated as follows: 

**Unstructured parameter evolution models**: A short time stochastic state space model (ST-CS method [7]) is employed with window length \(L = 1\,011\) samples. A Recursive ARMA (RARMA) model is also estimated via Recursive Maximum Likelihood (RML-ARMA) method with forgetting factor \(\lambda = 0.99905\). Three passes over the signal (forward, backward, forward) are used.

**Stochastic parameter evolution TARMA model**: ST-TARMA models are used with first and second order constraints. Estimation is based on Extended Least Squares (ELS) with three passes (forward, backward, forward) over the signal and an additional backward smoothing. The model with second order constraint is selected as best.

**Deterministic Functional Series TARMA**: Two Functional Series TARMA (FS-TARMA) methods are...
A conventional (referred to as method A) using trigonometric basis functions selected in the usual way \[2,7\], and a complete one (referred to as method B) in which the basis functions are decaying trigonometric with parameters estimated through a Separable Nonlinear Least Squares (SNLS) procedure \[9\]. An FS–TARMA(8,8)\[11,15,15\] model is identified in the first case and an FS–TARMA(8,8)\[7,7,7\] model is identified in the second.

A comparison of the estimated models in terms of achievable prediction accuracy (expressed by the Residual Sum of Squares, RSS, normalized by the Series Sum of Squares, SSS) and parsimony (in terms of number of estimated model parameters) is presented in Figure 3. The best predictive accuracy is achieved by the FS-TARMA model obtained by method A. The “frozen” TV PSDs based on the obtained models are compared to a baseline PSD (obtained via many stationary experiments) in Figure 4. Although all PSDs appear accurate, those of the SP–TARMA(8,8) and of the two FS–TARMA(8,8) models are best. In fact that of the FS–TARMA model obtained by method A is the best, with that obtained by method B following (exhibiting some problems in the intermediate mode).

4.2. Modelling of the El Centro (California) earthquake ground motion signal \[26\]

The modelling and simulation of the El Centro earthquake ground motion signal is considered. The accelerogram was recorded in California during the 1979 Imperial Valley earthquake (epicenter distance of 16 km). The sampling frequency is \(f_s = 50\) Hz and the signal length \(N = 1\,210\) samples (Figure 5).

Both RML-RARMA and FS-TARMA models are considered. An RML-ARMA(4,1) model with forgetting factor \(\lambda = 0.966\) and an FS-TARMA\(2,4\)\[8\] model with Haar basis functions are selected by each approach, respectively. Nevertheless, the predictive ability of the FS-TARMA model is considerably better than that of the RML-ARMA model (Figure 6).
4.3. Output-only identification of a wind turbine structural dynamics under normal operating conditions (also see [10])

The signal used in this case study was acquired on the tower of a NegMicon NM52/900 Horizontal Axis Wind Turbine located at a Rhodes, Greece, wind farm (also see [10]). The turbine consists of a 50 m tall tower and blades that are 25 m long. The whole system weights about 87 t on s. A piezoelectric accelerometer is placed on the tower, under the yaw system and about 48 m over the ground surface (Figure 7a). Vibration signals are acquired using a 4 channel portable DAQ DA-20 with 16 bit resolution and sampling frequency of \( f_s = 2.56 \text{ kHz} \). The signal is later down sampled to \( f_s = 160 \text{ Hz} \) (signal length \( N = 4 \times 800 \text{ samples} \); Figure 7b).

Fig. 7: Wind turbine structural dynamics identification:
(a) Schematic diagram of the experimental setup, and (b) the vibration acceleration signal analyzed.

The signal is modeled using both the SP-TAR and FS-TAR approaches. For SP-TAR modeling, models of orders 1 to 40 and constraint orders 1 and 2 are considered. An SP-TAR(31) model with constraint order 1 is selected as best according to the Bayesian Information Criterion (BIC) [5] and the Residual Sum of Squares (RSS) criteria. The “complete” FS-TAR identification method (method B with decaying trigonometric basis functions) is also used and models of orders 20 to 40 and functional subspace dimensionalities ranging from 1 to 11 are considered. An FS-TAR(31)\(_0\) model is thus selected. The resulting TV-PSDs for the two models are shown in Figure 8. The top plots show the TV-PSDs in the full frequency range, whereas the bottom plots show details in the frequency ranges 24 - 26 Hz and 34 - 42 Hz. The agreement between the two models is impressive. The presence of a cyclic modulation (with frequency of about 0.37 Hz) in the structural modes of both models is worth noting. This frequency practically coincides with that of blade rotation.

Fig. 8: Wind turbine structural dynamics identification. Estimated “frozen” TV-PSDs: (a) SP-TAR based estimate, (b) FS-TAR based estimate, (c) SP-TAR based estimate details (2D plots), and (d) FS-TAR based estimate details (2D plots).
5. Concluding remarks

A concise overview of parametric methods for non–stationary random vibration modeling and identification has been presented. The models – and corresponding identification methods - are suitable for describing the non–stationary vibration signal and the underlying TV structural dynamics. They are classified into three main groups: (i) unstructured parameter evolution, (ii) stochastic parameter evolution, and (iii) deterministic parameter evolution.

The class of deterministic parameter evolution methods utilizes Functional Series TARMA models, which are often physically motivated, and is capable of achieving very high accuracy and the most compact representations at the price of increased implementation complexity. In this context the newly developed Separable Nonlinear Least Squares (SNLS) based method (method B) seems capable of alleviating this drawback and significantly facilitating applications. The "frozen" time model analysis allows for direct insight into the underlying TV structural dynamics, while the "evolutive" analysis focuses more on the characteristics of the measured vibration signal.

Various models and corresponding identification methods were employed in three application case studies: (i) The output-only identification of a bridge-like structure with a moving mass, (ii) the modeling of the 1979 El Centro earthquake ground motion accelerogram, and (iii) the output-only identification of the structural dynamics of an operating wind turbine.

Further progress is expected on several topics, including a better understanding of the interconnections among the various methods, the further development of simple and effective methods for simultaneous model structure and parameter estimation (which is more critical in the FS–TARMA case), the resolution of algorithmic and model instability issues, and also on issues related to model analysis and interpretation.

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7. References


